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ON THE NUMERICAL SOLUTION OF SOME 2-D ELECTROMAGNETIC
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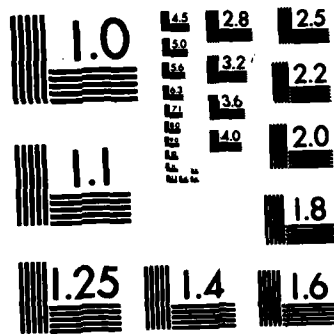
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ON THE NUMERICAL SOLUTION OF SOME 2-D
ELECTROMAGNETIC INTERFACE PROBLEMS BY
THE BOUNDARY COLLOCATION METHOD

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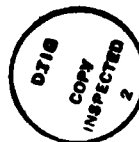
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ABSTRACT

The boundary collocation method is used to solve some 2-d interface problems of microwave heating. The choice of subspace of particular solutions, and the selection of collocation points are discussed. Numerical examples are presented.

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SIGNIFICANCE AND EXPLANATION

The boundary collocation method is a numerical method for solving boundary and interface problems for linear partial differential equations, for which a complete set of particular solutions is explicitly known. The method consists of the following steps:

- 1) Select a subspace of particular solutions.
- 2) Select a basis of the subspace.
- 3) Select a finite number of points on the boundary (or interface). The points are called collocation points. We require a finite linear combination of the basis functions to satisfy the boundary (or interface) conditions at the collocation points. We select sufficiently many points to uniquely determine the linear combination of basis functions.

How the selections 1) - 3) are done is important for the performance of the boundary collocation method. Guided by experience from solving a model interface problem for the Laplace operator, see [3], we describe a strategy for selecting subspace, basis and collocation points when applying the boundary collocation method to the numerical solution of 2-dimensional electromagnetic field problems of microwave heating. We present computed examples.

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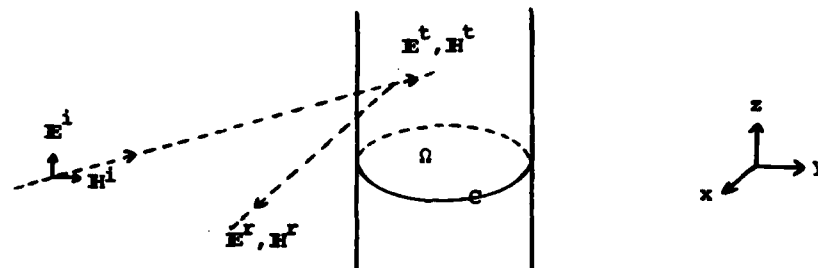
1. INTRODUCTION

In the present paper we describe how the boundary collocation method (BCM) can be applied to solve 2-dimensional electromagnetic field problems. We pay special attention to the selection of suitable subspaces of particular solutions, and to the allocation of collocation points. Our interest in the BCM stems from a recent comparison of integral equation methods and expansion methods for electromagnetic field computations reported by Bates [1]. He calls the boundary collocation method straightforward point matching (SPM), and states, [1], p. 619, "SPM is by far the most economical method from the points of view of programming effort and computer time." Bates goes on "But it [SPM] can only be used with confidence when C [the boundary curve] is such that the (external or internal) Rayleigh hypothesis is valid". The external Rayleigh hypothesis is the assumption that the reradiated field can be represented by a single expansion everywhere outside the object scattering the incident field. We, however, take a different view on the importance of the Rayleigh hypothesis. By allowing more general expansions than those in the comparison of Bates [1], and by letting the allocation of collocation points depend on the choice of (finite) expansion, the Rayleigh hypothesis seems to be of minor importance for the performance of the BCM.

Our approach is an experimental one. We generalize a strategy for selecting subspaces and collocation points, which has been found appropriate for 2-d interface problems for the Laplace and Poisson equations, see Reichel [3]. We apply this method to compute solutions to some interface problems of microwave heating. Section 2 contains the formulation of the interface problem, in section 3 we describe the BCM, and in the 4th section we present some computed examples.

2. FORMULATION OF THE INTERFACE PROBLEM

Let Ω be the cross section in the x,y -plane of a cylinder parallel to the z -axis. Denote the smooth boundary curve of Ω by C , and let Ω_c be the exterior of C in the x,y -plane.



$$\mathbf{E}^i = (E_x^i, E_y^i, E_z^i) \quad \text{incident electric field}$$

$$\mathbf{H}^i = (H_x^i, H_y^i, H_z^i) \quad \text{incident magnetic field}$$

$$\mathbf{E}^r = (E_x^r, E_y^r, E_z^r) \quad \text{reflected electric field}$$

$$\mathbf{H}^r = (H_x^r, H_y^r, H_z^r) \quad \text{reflected magnetic field}$$

$$\mathbf{E}^t = (E_x^t, E_y^t, E_z^t) \quad \text{transmitted electric field}$$

$$\mathbf{H}^t = (H_x^t, H_y^t, H_z^t) \quad \text{transmitted magnetic field}$$

Figure 2.1

We assume that the cylinder is homogeneous and surrounded by free space. Let the incident electric field be a plane wave with only the z -component nonvanishing. Then the problem is 2-dimensional, and can be expressed as an interface problem in the plane for the Helmholtz operator, see Figure 2.2

$$\begin{array}{ccc}
 \Delta u_1 + c_1^2 u_1 = 0 & & \Delta u_2 + c_2^2 u_2 = 0 \\
 \Omega & & \Omega_c
 \end{array}$$

c_1, c_2 constants, $c_1 \in \mathbb{C}$, $c_2 \in \mathbb{R}$.

Figure 2.2

We will refer to this figure later. Next we give expansions, which satisfy Maxwell's equation, for the electric and magnetic field. The time dependence enters as factors $e^{i\omega t}$, which can be cancelled. We may assume E^i is of amplitude 1. The components of E^i then are

$$(2.1) \quad E_z^i = e^{i\beta_2 x}, \quad E_x^i = E_y^i = 0, \quad \beta_2 = (\epsilon_2 \mu_2)^{1/2} \omega,$$

where $\epsilon_2 = \frac{1}{36\pi} \cdot 10^{-9}$ F/m and $\mu_2 = 4\pi \cdot 10^{-7}$ H/m are the dielectric constant and permeability of free space. The components of H^i are, see Wait [4],

$$(2.2) \quad H_y^i = \frac{\beta_2}{\omega \mu_2} e^{i\beta_2 x}, \quad H_x^i = H_z^i = 0.$$

The interface condition that the tangential components of the electric and magnetic fields should be continuous across the interface immediately gives us

$$(2.3) \quad E_x^r = E_x^t = 0, \quad E_y^r = E_y^t = 0, \quad H_z^r = H_z^t = 0.$$

We assume the origin belongs to Ω and seek E_z^t as a linear combination of finitely many terms $J_n(\beta_1 r) e^{-in\theta}$, where (r, θ) are polar coordinates in the plane, J_n is the n th order Bessel function, and $\beta_1 = (\epsilon_1 \mu_1)^{1/2} \omega$. μ_1 is the permeability of the cylinder, and $\epsilon_1 = \kappa \epsilon_2$ is the complex valued dielectric constant for the cylinder with

$$(2.4) \quad \kappa^{1/2} = \sqrt{\frac{1}{2} (\kappa_1 + \sqrt{\kappa_1^2 + \kappa_2^2})} + i \sqrt{\frac{1}{2} (-\kappa_1 + \sqrt{\kappa_1^2 + \kappa_2^2})},$$

κ_1, κ_2 real > 0 . In the numerical examples of section 4, the cylinder is assumed to be of metal. Then we let $\nu_1 = \nu_2$, and from Ohlsson and Risman [2], we obtain

$$(2.5) \quad \kappa_1 = 36, \quad \kappa_2 = 16.$$

This corresponds to coefficients $c_1 = 0.33 + i0.07$ and $c_2 = 0.05$ in Figure 2.2. Thus, we seek E_z^t in the form

$$(2.6) \quad E_z^t = \sum_{n=-N}^N a_n J_n(\beta_1 r) e^{-in\theta}, \quad a_n \in \mathbb{C},$$

and for H^t one obtains, see Wait [4],

$$(2.7) \quad H_x^t = \sum_{n=-N}^N a_n \left(\frac{i}{\omega \mu_1} J'_n(\beta_1 r) \beta_1 e^{-in\theta} \sin\theta + \frac{n}{\omega \mu_1} J_n(\beta_1 r) e^{-in\theta} \frac{\cos\theta}{r} \right)$$

$$(2.8) \quad H_y^t = - \sum_{n=-N}^N a_n \left(\frac{i}{\omega \mu_1} J'_n(\beta_1 r) \beta_1 e^{-in\theta} \cos\theta - \frac{n}{\omega \mu_1} J_n(\beta_1 r) e^{-in\theta} \frac{\sin\theta}{r} \right).$$

$J'_n(\alpha)$ denotes $\frac{d}{d\alpha} J_n(\alpha)$.

We now turn to the reflected field. Let $H_n(r)$ be the Hankel function $H_n(r) = J_n(r) - iY_n(r)$, with derivative H'_n . Let (x_j, y_j) , $j = 1(1)p$, be a finite number of distinct points in the interior of Ω , and let (r_j, θ_j) be polar coordinates with respect to (x_j, y_j) , see Figure 2.3.

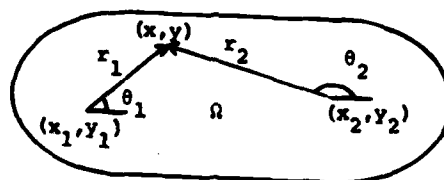


Figure 2.3

In section 3 we describe how to select the (x_j, y_j) . Given the (x_j, y_j) we seek E_z^r in the form

$$(2.9) \quad E_z^r = \sum_{j=1}^P \left(\sum_{n=-N_j}^{N_j} b_{nj} H_n(\beta_2 r_j) e^{-in\theta_j} \right), \quad b_{nj} \in \mathbb{C}.$$

For the reflected magnetic field, we get, see Wait [4],

$$(2.10) \quad H_x^r = \sum_{j=1}^P \left(\sum_{n=-N_j}^{N_j} b_{nj} \left(\frac{i}{\omega \mu_2} H'_n(\beta_2 r) \beta_2 e^{-in\theta} \sin\theta + \frac{n}{\omega \mu_2} H_n(\beta_2 r) e^{-in\theta} \frac{\cos\theta}{r} \right) \right)$$

$$(2.11) \quad H_y^r = - \sum_{j=1}^P \left(\sum_{n=-N_j}^{N_j} b_{nj} \left(\frac{i}{\omega \mu_2} H'_n(\beta_2 r) \beta_2 e^{-in\theta} \cos\theta - \frac{n}{\omega \mu_2} H_n(\beta_2 r) e^{-in\theta} \frac{\sin\theta}{r} \right) \right)$$

We have already stated and made use of the interface condition. For later reference, we give the equations obtained from the interface condition for the nonvanishing components of the electric and magnetic fields. On the boundary curve C , the interface conditions are

$$(2.12) \quad E_z^i + E_z^r = E_z^t$$

$$(2.13) \quad (H_x^i + H_x^r) t_x + (H_y^i + H_y^r) t_y = H_x^t t_x + H_y^t t_y,$$

where $t = (t_x, t_y)$ is a unit tangential vector to the curve C .

3. THE BOUNDARY COLLOCATION METHOD

We require the interface conditions (2.12) and (2.13) to be satisfied in a finite number of points, collocation points, distributed on C . This gives a system of linear equations for the coefficients a_n, b_{nj} of the expansions (2.6)-(2.11). Numerical experiments indicate that

- 1) the selection of subspace for the reflected field, i.e. the selection of (x_j, y_j) , $j = 1(1)p$, see Figure 2.3, is important for the performance of the method.
- 2) the subspace and collocation points should not be selected independently.
- 3) the expansion (2.6)-(2.8) for the transmitted field is generally adequate for cylinders which are not pronouncedly nonconvex. If a large approximation error is obtained, the addition of few extra basis functions with a singularity in the finite plane may reduce the approximation error substantially, as is demonstrated in ex. 4.4, section 4.

The following scheme for choice of subspace and collocation points is an adaption of the method described in Reichel [3] for the numerical solution of an interface problem for the Laplace and Poisson equations.

- a) Let $z_j := x_j + iy_j$. Select the z_j in Ω , not too close to the boundary, such that $\prod_{j=1}^p |z - z_j|^{1/p}$ is near constant for z traversing the boundary curve C . We either guess appropriate z_j or use a method described in [3] for their allocation. A comparison with approximation problems for harmonic polynomials and harmonic rational functions leads us to the following choice of N and M_j . For a given N , let $M := \sum_{j=1}^p M_j + p$ satisfy $M = N + 1$. Select the M_j so that $\max_j M_j - \min_j M_j$ is as small as possible. We assume throughout the remainder of this paper that $N > p - 1$. Then $M_j > 0 \quad \forall j$.

- b) The density function for the collocation points σ is obtained by solving the integral equation

$$(3.1) \quad q + \int_C \ln|z - \zeta| \sigma(\zeta) |d\zeta| = \frac{1}{2p} \sum_{j=1}^p \ln|z - z_j|, \quad z \in C$$

$$\int_C \sigma(\zeta) |d\zeta| = 1$$

for $\{\sigma, q\}$, where q is a constant. Determine $M + N + \frac{1-p}{2}$ collocation points τ_j by solving

$$(3.2) \quad \int_{\tau_j}^{\tau_{j+1}} \sigma(\zeta) |d\zeta| = (M + N + \frac{1-p}{2})^{-1}, \quad j = 1(1)(M + N + \frac{1-p}{2} - 1), \quad \tau_1 \text{ arbitrary.}$$

If p is even, replace $M + N + \frac{1-p}{2}$ by $M + N - \frac{p}{2}$.

Integration in (3.2) is understood to be along C in the positive sense. (3.1) and (3.2) are conveniently solved as described in [3]. We note that σ does not have to be determined to very high accuracy.

c) At each collocation point τ_j , we require both (2.12) and (2.13) to be satisfied. For p odd, this gives $2M + 2N + 1 - p$ complex linear equations for the same number of coefficients a_n, b_{jn} . In case p is even, we only get $2M + 2N - p$ complex linear equations, and require now also $a_{-M} = 0$. Since the basis functions might not form a Chebyshev system on the set of collocation points, we solve the linear system by singular value decomposition, and have the option to discard singular values of the same order of magnitude as round-off errors.

4. NUMERICAL EXAMPLES

For a few cylinders we have computed the power density $|E|^2$ generated by an incident plane transverse magnetic waves of 2450 MHz. The power density is of interest when studying microwave heating, see [2]. The cylinders have the dielectric properties of meat, see (2.4), and can be thought of as meat loaves or sausages. Symmetry properties of the cylinders below have not been used in the numerical computations. We measure the error in the interface condition of the computed solution with the norm

$$\|f\| := \max \left\{ \max_{(x,y) \in C} |\operatorname{Re} f(x,y)|, \max_{(x,y) \in C} |\operatorname{Im} f(x,y)| \right\},$$

All computations were carried out on a DEC-10 computer in single precision, i.e. with 8 significant digits.

Ex. 4.1. Let C be the ellipse $\{(x,y) : (\frac{x}{40})^2 + (\frac{y}{20})^2 = 1\}$, with x,y in mm. Let $p = 5$, and select the points (x_p, y_p) as the zeros of the 5th degree Chebyshev polynomial of the 1st kind for the interval between the foci at $\pm 10\sqrt{12}$ of the ellipse. Let $M = 16$ and $N = 15$. This gives 58 coefficients to determine, and we use the collocation points $(40 \cos(2\pi \frac{k-1}{29}), 20 \sin(2\pi \frac{k-1}{29}))$, $k = 1(1)29$. One can show that these points are very close to those obtained from (3.1), (3.2). Figure 4.1 shows Ω with level curves of the power density. The level curves show levels $k = 0.05$, $k = 1, 2, \dots$. The value of $|E|^2$ is shown at some points along the major axis of the ellipse. (The points in the figure are decimal points, and do not indicate location.)

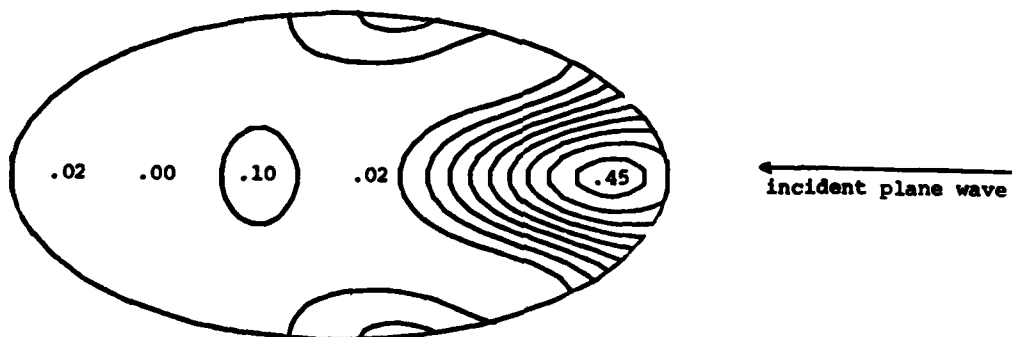
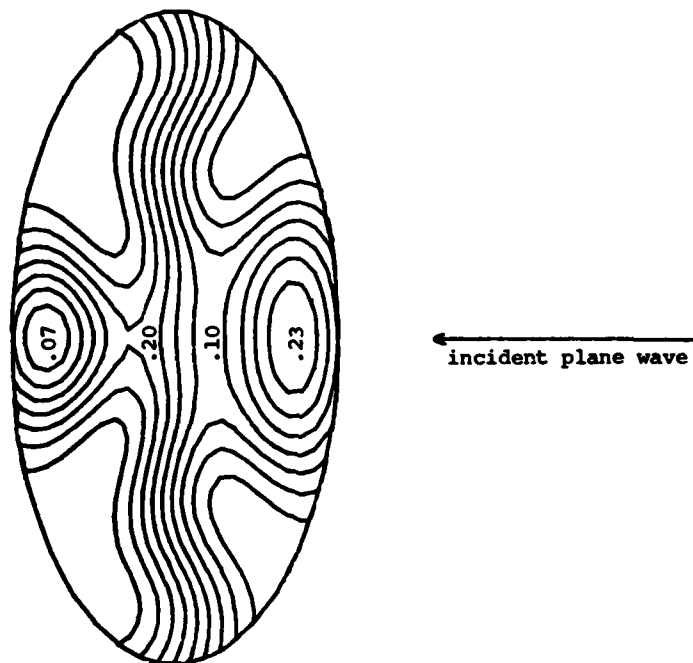


Figure 4.1

For the computed reflected and transmitted electric field we measure the maximum discrepancy in the interface condition over C . We obtain $\|E^i + E^r - E^t\| = 1 \cdot 10^{-3}$. In this and all the following examples the real and imaginary part of $E^i + E^r - E^t$ are of the same order of magnitude. The magnetic fields are of much smaller magnitude than the electric ones, and in all examples the relative error in the interface condition for the magnetic field is of the same order of magnitude as the relative error for the electric field.

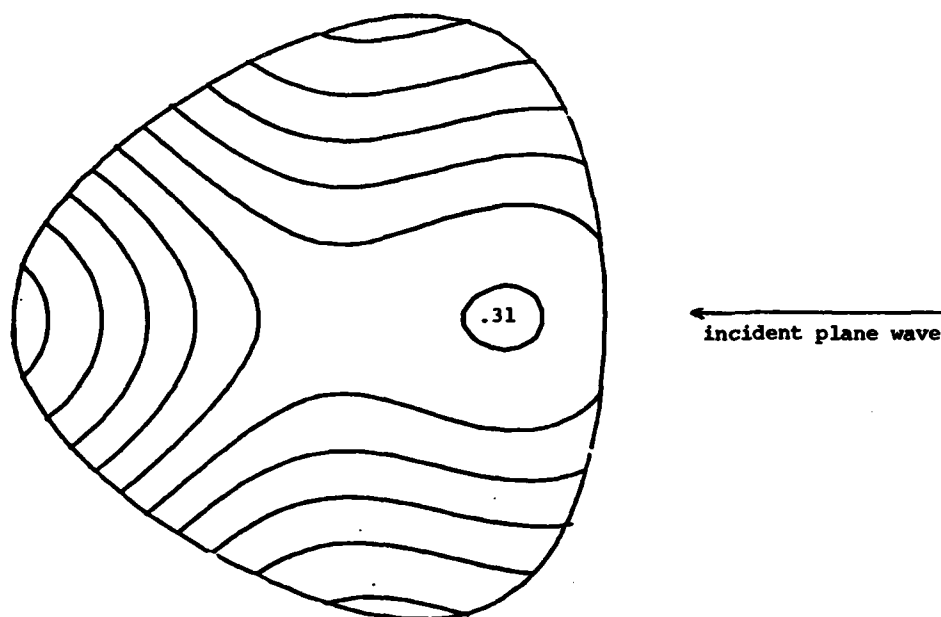
Ex. 4.2



We change the direction of the incident field, and increase the number of terms in the expansions, compared to ex. 4.1. Now $N = 23$, $M = 24$. This gives

$\|E^i + E^r - E^t\| = 2 \cdot 10^{-4}$. The level curves display levels $k \cdot 0.03$, $k = 1, 2, \dots$.

Ex. 4.3. Let C be the curve $\{z(t) = 6e^{it} + 0.6e^{-2it}, 0 \leq t < 2\pi\}$, $z = x + iy$ in mm. Let $p = 3$ and $(x_1, y_1) = (5.26, 0)$, $(x_2, y_2) = (-2.63, 4.50)$, $(x_3, y_3) = (-2.63, -4.50)$. Select $N = 15$, $M = 16$. Then $\|E^i + E^r - E^t\| = 4 \cdot 10^{-4}$. Figure 4.3 shows level curves for levels $k \cdot 0.05$, $k = 1, 2, \dots$.



Ex. 4.4. Let C be the superellipse $\{(x,y) : x^4 + y^4 = 9\}$, x,y in mm. Let $p = 5$ and (x_j, y_j) be the points $(0,0)$, $(\pm \frac{9}{\sqrt{2}}, \pm \frac{9}{\sqrt{2}})$. Select $N = 22$, $M = 23$ and augment the expansion for the transmitted field by 4 functions $H_0(\beta_j \rho_j)$, $j = 1(1)4$, where ρ_j is the distance between (x,y) and $9 \cdot 8^{1/4} e^{i\pi/2 j}$. These augmented basis functions are singular exterior to C at points where the Schwarz function for C is singular, see [3]. We allocate 48 collocation points with (3.1), (3.2). The computed solution satisfies

$$\|E^i + E^r - E^t\| = 3 \cdot 10^{-3}$$

The inclusion of the extra basis functions decreases the error by a factor 50. Figure 4.4 shows level curves for levels $k \cdot 0.05$, $k = 1, 2, \dots$.

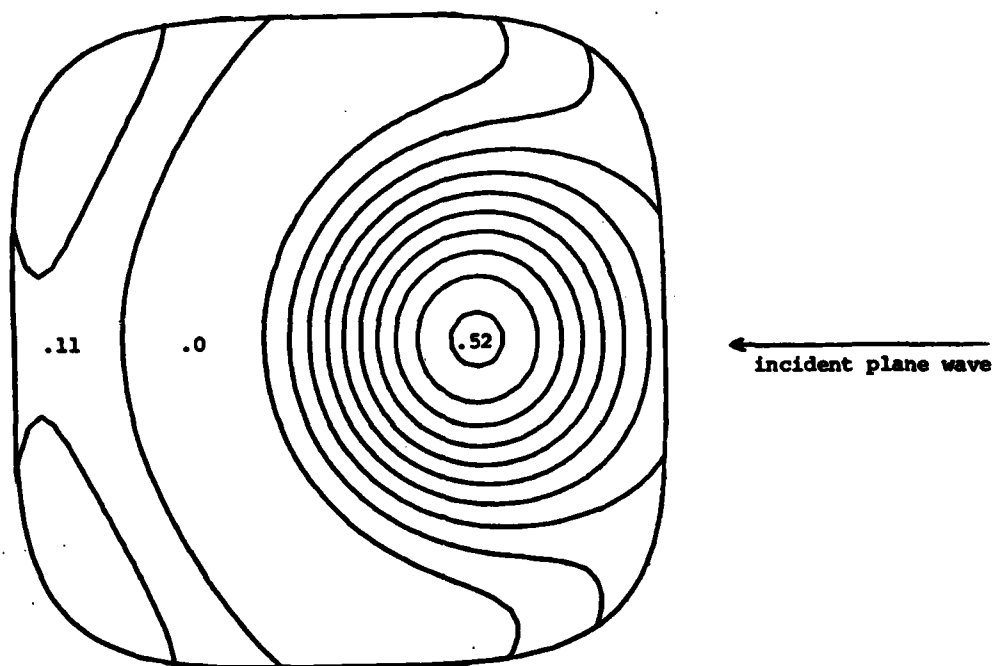


Figure 4.4

Acknowledgement

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